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High speed stereo-vision study of laminated glass fragmentation upon impact

Corentin LE GOURRIÉREC^{a,b,*}, Xuyang CHANG^a, Bastien DURAND^a,
Richard VILLEY^c, Benoît VOILLOT^b, Xavier BRAJER^b, Stéphane ROUX^a

^a*Université Paris-Saclay, CentraleSupélec, ENS Paris-Saclay, CNRS, LMPS -
Laboratoire de Mécanique Paris-Saclay, 4 Avenue des
Sciences, Gif-sur-Yvette, 91190, France*

^b*Saint-Gobain Research Paris, 39 Quai Lucien Lefranc, Aubervilliers, 93300, France*

^c*Saint-Gobain Research Compiègne, 1 rue de Montluçon, Thourotte, 60150, France*

Abstract

The different stages of an impact on laminated glass are difficult to observe and quantify but are essential for numerical model validation. To assess the performance of this composite to low-velocity (~ 10 m/s) high-mass impacts, dynamical impact tests, inspired by ball-drop tests, are performed, equipped with multiple high-speed cameras. From the set of images, a continuous 3D shape reconstruction of the deformed laminated glass over time is performed using Stereo-Digital Image Correlation (S-DIC). Significant brightness and contrast variations captured by two cameras due to large rotations of glass fragments and large strain of the polymer interlayer between fragments render

*Corresponding author

Email addresses: corentin.legourrierrec@saint-gobain.com (Corentin LE GOURRIÉREC), xuyang.chang@ens-paris-saclay.fr (Xuyang CHANG), bastien.durand@ens-paris-saclay.fr (Bastien DURAND), richard.villey@saint-gobain.com (Richard VILLEY), benoit.voillot@saint-gobain.com (Benoît VOILLOT), xavier.brajer@saint-gobain.com (Xavier BRAJER), stephane.roux@ens-paris-saclay.fr (Stéphane ROUX)

the use of S-DIC difficult as the impactor penetrates deeper into the plate. To circumvent this problem, an axisymmetric regularization is introduced which has the advantage of not requiring prior knowledge of the mechanical behavior for the tested specimen. It is shown that it offers a more extended period of time (as compared to regular S-DIC) over which a trustful analysis can be performed. The time evolution of the fragmented laminated glass and the density of cracks, both available from our analysis, are key ingredients to elaborate quantitative modeling of the dynamic performance of the laminate. *Keywords:* stereo-Digital Image Correlation, stereo-vision, laminated glass, low-velocity impact, hard-body impact.

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Declaration of Competing Interest

The authors declare that they have no known competing interests.

Authors contribution

All authors planned the experiments, the experimental strategy and discussed the results. CLG, RV, BV carried out the experiments. CLG per-

formed the S-DIC measurements with Correli 3.0 and implemented the axisymmetric regularization. XC implemented and performed the numerical calibration of the stereo-vision system. CLG, SR, XC, BD, RV wrote the paper.

1. Introduction

Laminated glass, a composite made of two or more different glass sheets interleaved with thin polymeric layers, is widely used for building or automotive applications. It has been engineered to meet a wide range of demanding properties: transparency, stiffness, low weight, acoustic insulation, and, most importantly, its ability to ensure the safety of people upon shocks or impacts due to the residual resistance offered by the polymeric layers following the fracture of the glass sheets. Therefore, research topics in the resistance to impacts focus on the designing of new laminated glass plates while respecting the criteria of personal protection, which remains the major characteristic of such glazing. Characterizing and quantifying the mechanical response of laminated glass during a low-velocity impact, especially structural degradation and integrity is a critical issue for manufacturers of laminated glass, who must validate certain norms to sell their production ([European Norm 2000; 2003](#)). Such impacts can be met when a pedestrian is hit by a car ([Chen et al. 2016](#)), when people fall from their height onto a laminated glazing ([Huang et al. 2023](#)), or if hand tools are used for a burglary ([Schuster et al. 2020](#)), for instance. Each of these loading cases has a velocity in the 1 to 10 m/s range. During such dynamic load, just after contact, cracks are initiated and propagated in the glass plies. Thanks to a finely adjusted adhesion between glass and polymer interlayer, with large stretching capacity and toughness ([Martin et al. 2020](#)) – for example made of polyvinyl butyral (PVB) –, the newly-fragmented plate can accommodate large out-of-plane displacements via the glass fragments delamination from the stretched membrane, dissipating a large amount of the energy. Adhesion between the layers is also tuned –

26 regarding the polymer layer thickness – to prevent large splinters from de-
27 taching then causing injuries, and avoid projectile penetration through the
28 structure (Pelfrene et al. 2016).

29 During a typical fragmentation test with the *cannon* (Nourry 2005), the
30 half-sphere apex impacts the glass surface at that initial velocity v_0 and
31 with a normal incidence. In the first microseconds after impact, the lami-
32 nate behaves elastically for the most part, while a small zone of the first ply
33 right under the impactor head is crushed under intense multiaxial loading.
34 Depending on the impact velocity and the sample strength, the first radial
35 cracks may appear from the plate center within a few microseconds to a few
36 hundreds of microseconds. As the plate bends, the radial cracks extend and
37 sometimes become more numerous, while orthoradial cracks are also formed.
38 More and more cracks are created during the whole impact duration. When
39 the fragmentation is well-advanced, the stretching (and possibly tearing out)
40 of the PVB interlayer dissipates a large amount of the impact energy as the
41 impactor stroke increases. This dissipation occurs at the PVB bridge loca-
42 tions that connect all the glass fragments (Seshadri et al. 2002). The initial
43 projectile velocity plays an important role in the laminated glass behavior to
44 impact. Two failure modes can be distinguished as introduced by Li et al. (Li
45 et al. 2023): the *punching failure* which involves intense shear loading in the
46 membrane structure of a densely fragmented plate, and the *bending failure*
47 with fewer cracks formed and much more bending energy stored in the glass
48 – in compression – and in the interlayer – in tension (Li et al. 2023).

49 Many experimental studies have been carried out to better understand
50 the mechanics of laminated glass under impact. Particularly, the semi-

51 rigid or head-form impact on windshields is at stake: Yu et al. tested
52 different locations on windshields and analyzed the impact acceleration ver-
53 sus the crack pattern evolution on the fragmented plate (Yu et al. 2017).
54 Yunqi Li et al. compared soda-lime and tempered glass performances (Li
55 et al. 2019). Dong Li et al studied the dissymmetry of the glass layers regard-
56 ing impact performances (Li et al. 2023). Also, Nassr et al. introduced a link
57 between the impact energy and the size of the damage on the sample (Nassr
58 et al. 2021). Finally Angelides et al. discussed the time-temperature depen-
59 dence of the interlayer yield stress (Angelides et al. 2021) and its influence on
60 the post-fractured laminated glass bending behavior (Angelides et al. 2022).
61 A large number of numerical models has been proposed to reproduce the
62 crack pattern and to simulate the impact resistance of the laminated glass
63 regarding the previous experimental results: using Rankine criterion for glass
64 breakage (Pyttel et al. 2011; Alter et al. 2017; Fernandes et al. 2021), com-
65 bined with cracking and element deletion (Hála et al. 2023), or a cohesive zone
66 model between the layers (Chen et al. 2016; Shahriari and Saeidi Googarchin
67 2020). Peridynamic solution is also used to describe the crack pattern (Lai
68 et al. 2018), the PVB/glass adhesion strength (Samieian et al. 2019), and the
69 glass cracking plus interfacial debonding (Ma et al. 2022). The simulations
70 generally give plausible results that are compared to post-fractured analyses
71 of the laminated glass damage. However, it is more difficult to verify the
72 consistency of the model at different times of a test because in-situ measure-
73 ments on laminated glass under impact are very challenging, notably due to
74 the structure change when cracks develop. Indeed, the relative movement of
75 the fragments during the tests is seldom studied, the randomness and very

76 fine fragmentation requiring accurate measurement in time and space. This
77 leads only to post-impact analyses of fragmented samples after rigid-body
78 blunt impacts on laminates in the literature. These above-mentioned exper-
79 imental works (Yu et al. 2017; Li et al. 2019; 2023; Nassr et al. 2021) have
80 brought significant characteristics of the dynamic behavior of laminated glass
81 through a few key quantities (*e.g.*, acceleration of the projectile, strain gauge
82 measurement on the sample, high-speed mono-vision). Yet, these – mainly
83 local – measurements do not provide a global quantitative picture of the
84 whole plate, rendering the comparisons with the numerical modeling difficult
85 and non-exhaustive.

86 The low-velocity rigid body impact on burglar-proof laminated glass as
87 described in standard *EN 356* (European Norm 2000), corresponds to a series
88 of three successive 4 kg steel ball drops from a height in the range of 1.5 m to
89 9 m, on a 1 m² laminate. This standard aims to prevent perforation, which is
90 also important for protecting people against falls. To better apprehend this
91 test, Nourry (Nourry 2005) designed an impact-test device, named *cannon*,
92 that can operate on smaller laminated glass samples in a controlled and
93 quantified way.

94 In a general context, the experimental tools to characterize the impact
95 response of either impactor or laminated glass can be classified into three
96 major categories:

- 97 1. Direct measurements on the projectile/impactor (Yu et al. 2017; Li
98 et al. 2019; Wang et al. 2020; Li et al. 2023).
- 99 2. Detection of wave propagation on the glass surface via multiple strain
100 gauges or accelerometers glued at different radial distances from the

101 impact location (Wang et al. 2020; Nassr et al. 2021).
102 3. Kinematic field measurements, combined with high-speed cameras cap-
103 turing the *in-plane* cracking of the glass plies (Del Linz et al. 2017;
104 Mohagheghian et al. 2017; Wang et al. 2021).

105 Compared to these other techniques, *Stereo-Digital Image Correlation* (Sut-
106 ton et al. 2009) (S-DIC) is a non-contact, non-destructive, robust, and pre-
107 cise optical method to access 3D shape and kinematic information (motion
108 and deformation) of large structures (Dufour et al. 2015b; Genovese and
109 Sorgente 2018). For instance, the S-DIC is used to analyze blast loading
110 experiments (Hooper et al. 2012) or bird strike experiments (Mohagheghian
111 et al. 2018) and validate associated numerical models by comparing the de-
112 formed shape of the observed surface to the simulated one. Moreover, by
113 expressing S-DIC into a global Finite Element based formulation, the virtual
114 model (*e.g.*, CAD (Dufour et al. 2015a) or Finite Element mesh (Pierré et al.
115 2017; Berny 2020; Colantonio et al. 2020)) can be flexibly deformed to fit
116 seamlessly with the real deformed 3D shape. This allows for shape charac-
117 terization and kinematic field measurement of complex 3D objects, even for
118 large deformation configurations.

119 The focus of the current study is mainly experimental via analyzing a
120 *cannon* test. It is proposed to measure the glass shards formation and kine-
121 matics. Direct kinematic measurements on the sample will be compared to
122 the kinematics of the impactor. The experimental set-up and the developed
123 axisymmetric S-DIC code will be presented. The novelty of this paper lies
124 in the use of a DIC regularization method exploiting the axisymmetry of
125 the test to counter the numerous and severe artifacts that can occur during

126 the test, such as the change in brightness and contrast or the discontinuities
127 (*i.e.*, cracks) on the measured surface.

128 The paper is organized as follows: section 2 introduces the *cannon* de-
129 vice and its stereo camera setup. Section 3 addresses the details of spatial-
130 regularization implementation into an already existing global S-DIC code.
131 The obtained results are presented and discussed in section 4. The first
132 3 ms of an impact are analyzed, including kinematic fields and crack pattern
133 measurements, and the regularization is further discussed. Finally, section 5
134 draws some conclusions addressing, in particular, the consequences of the
135 experimental observations for a numerical model of laminated glass dynamic
136 behavior.

137 2. Experimental device and material

138 2.1. Experimental device

139 Initially developed by E. Nourry (Nourry 2005), the *cannon* test device
140 aims at reproducing hard impacts (especially blunt and rigid objects) on
141 laminated glass in a controlled way via a low-velocity/large-mass impact.
142 The set-up allows the impact experiments to replicate the desired conditions
143 by controlling the impactor velocity and the laminated glass sample charac-
144 teristics (*e.g.*, layers thickness, previous damage). This concept was further
145 explored through later studies, *e.g.*, Decourcelle's (Decourcelle 2011), and re-
146 sults were analyzed using tensile tests on pre-cracked laminated glass samples
147 as a model describing impactor kinetic energy dissipation via delamination
148 and tearing of the polymer ply (Elzière 2016). A key feature of this set-up
149 is that it offers the measurement of the impactor head motion via a laser

150 telemeter at the high rate of 30 kHz, from which velocity (using finite differ-
151 ence) and kinetic energy can be estimated assuming the impactor head to be
152 rigid (non-deformable).

153 This device allows testing different specimen thicknesses of the plies and
154 materials to compare quantitatively their impact performances, with a good
155 reproducibility and explore a range of impact velocity from 2 to 15 m/s.

156 The impacted samples are square-shaped plates of 300×300 mm²: this
157 surface is about 10 times smaller than that of ball-drop tests but corresponds
158 to the size of the damaged surface during standard *EN 356* impacts. During
159 the few tens of milliseconds of an impact experiment the boundary conditions
160 — namely the fastening of the plate, the impactor geometry, and the center-
161 to-edge distance — of the laminated glass plate drives the cracking of the
162 glass and, hence, the PVB stretching and tearing. Here, the fastening of
163 the glass is well-controlled and reproducible: two polished steel rings, with
164 curvature radius of 1 mm in contact with the glass — large enough to avoid
165 any sharp indentation —, and tightened by 4 screws at 10 N·m. Additionally,
166 Decourcelle (Decourcelle 2011) highlighted mechanical wave propagation on
167 the glass sample, provoking the first orthoradial cracks, are not significantly
168 dependent on the sample size (Decourcelle 2011), justifying the use of smaller
169 samples (*i.e.*, 300×300 mm²) than in the norm *EN 356* (*i.e.*, 1100×900 mm²).
170 However it has an influence on more advanced stages of the test, where the
171 finite strain regime and large out-of-plane bending is at stake.

172 As schematized in Fig 1, a projectile is set in motion via a compressed air
173 shot, and hits the cylindrical end of the impactor. The impactor is composed
174 of a cylindrical body, guided in a lubricated bronze barrel, whose head is a

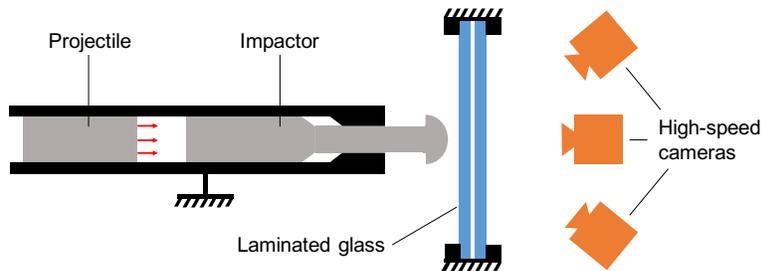


Fig 1: Scheme of a *cannon* experiment. The impactor and projectile are guided in the barrel. The impactor head finally hits the laminated glass with a normal impact. If the laminated glass is completely pierced, the impactor is stopped by an arrest cone in the barrel.

175 blunt half-sphere made of steel to replicate the contact surface of the ball-
 176 drop tests. The whole impactor weight is designed to be 4.3 kg. A telemeter
 177 measures via laser triangulation method, the hemisphere displacement at
 178 30,000 Hz, with a 3 μm uncertainty. These high rate measurements allow
 179 to evaluate axial accelerations and velocities through finite difference. The
 180 impactor initial speed is $v_0 = 8.28 \text{ m/s}$ in the reported experiment. It is a
 181 median value of the impact velocity range reached on the device, that is also
 182 close to the P2A classification of the norm *EN 356*. In accordance with the
 183 laminate design, in the present case, the velocity is set to avoid perforation
 184 at the end of the impact.

185 During a typical fragmentation test with the *cannon*, the half-sphere apex
 186 impacts the glass surface at that initial velocity v_0 and with a normal inci-
 187 dence. In the first microseconds after impact, the laminate behaves elastically
 188 for the most part, while a small zone of the first ply right under the impactor
 189 head is crushed under intense multiaxial loading. Depending on the impact
 190 velocity and the sample strength, the first radial cracks may appear from the

191 plate center within a few microseconds to a few hundreds of microseconds.
192 As the plate bends, the radial cracks extend and sometimes become denser,
193 while orthoradial cracks are also formed. More and more cracks are created
194 during the whole impact duration. When the fragmentation is well-advanced,
195 the stretching (and possibly tearing out) of the PVB interlayer dissipates a
196 large amount of the impact energy as the impactor stroke increases. This
197 dissipation occurs at the PVB bridge locations that connect all the glass
198 fragments (Seshadri et al. 2002).

199 The interlayer thickness can influence the time to tearing as a competi-
200 tion between delamination and stretching occurs: for high strain rates, less
201 time is given for delamination initiation leading to a very large deforma-
202 tion of the interlayer that ends up tearing prematurely. Therefore, a thicker
203 interlayer may sustain higher impact regimes as the delamination is given
204 more time to initiate (Morison 2007; Hooper 2011; Angelides et al. 2019).
205 In the present experiment, the laminated glass sample is composed of two
206 4 mm thick Planiclear[®] soda-lime glass plies, and a 0.76 mm PVB interlayer
207 Saflex[®] RB41. The laminates are assembled in a clean room, degassed using
208 an oven and a calender adhesion fully develops in a last stage where the lam-
209 inate is placed in an autoclave, where the temperature is set to about 130°C
210 and pressure to 13 bars for several hours.

211 2.2. Stereovision setup

212 Cameras, shown on Fig 2, are used to record the details of the dynamic
213 fracture. The first camera (2) is a Shimadzu[®] ‘HPV-X2’ that captures the
214 crack initiation at 100,000 Hz with a 400×250 pixel definition. The camera
215 axis is coincident with the *cannon* z -axis. Two synchronized (at ± 20 μ s)

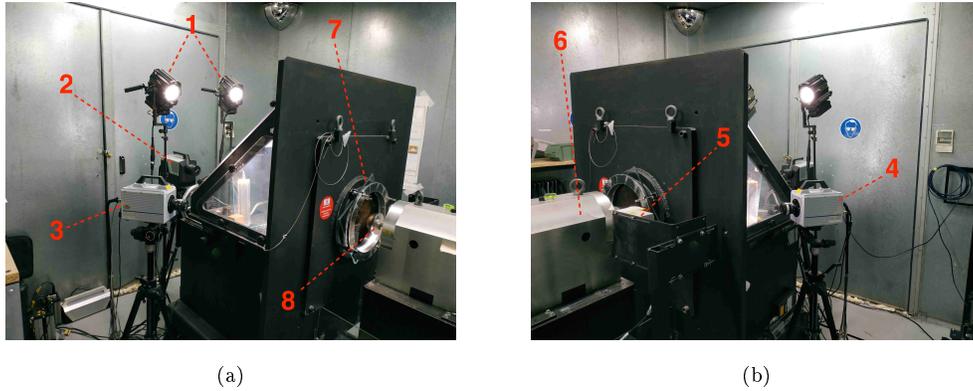


Fig 2: The stereo camera setup: **1** The 400W spotlights. **2** Shimadzu[®] HPV-X2. **3** Right-view Photron[®] SA-5 (24 mm lens). **4** Left-view Photron[®] SA-5 (24 mm lens). **5** Telemeter. **6** Barrel box. **7** Laminated glass sample clamped in a circular window. **8** Impactor half-sphere.

216 Photron[®] ‘SA-5’ cameras (3 & 4) are positioned on each side of the device,
 217 monitored at 30,000 Hz. All images are 8-bit encoded (*i.e.*, the gray level of
 218 each pixel may vary from 0 to 255). The definition of the two lateral cameras
 219 decreases as the acquisition rate increases. In this experiment setting, their
 220 total definition is 640×376 pixels. Moreover, due to the very short exposure
 221 time ($\leq 30 \mu\text{s}$) of the fast cameras, two 400 W spotlights are oriented towards
 222 the sample to provide enough light with negligible PVB heating. Indeed, due
 223 to the short exposure time it is not expected that temperature effects will
 224 influence the material properties of the interlayer.

225 The incident angle between the camera axes and the *cannon* principal
 226 axis varies on each side from 30° to 45° depending on the remaining space
 227 around the large test bench. All cameras are triggered using the same photo-
 228 electric sensor that detects the projectile presence a few milliseconds before
 229 its contact with the impactor, with a 40 kHz sampling rate.

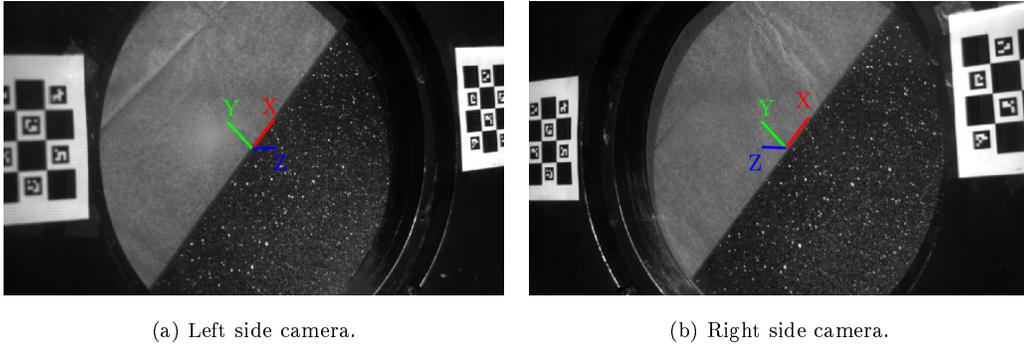


Fig 3: Initial view of the two lateral cameras. World coordinate system projected on the images, validating the calibration of the presented stereoscopic camera system.

230 Observing the back surface of the laminated glass at high temporal res-
 231 olution has two motivations: first, the measurement of surface deformation,
 232 and second, the characterization of glass plies fragmentation. Unfortunately,
 233 these two goals are mutually exclusive. Indeed, the kinematics of the back
 234 surface can be obtained from S-DIC, provided a surface marking with a
 235 speckle pattern is deposited on the back face. In turn, the formation of
 236 cracks is mostly hidden due to the opaque paint presence. To circumvent
 237 these difficulties, a black and white speckle pattern is applied on the back
 238 surface of the laminated glass sample, covering the right-bottom part of the
 239 laminated glass, providing sufficient Gray Level (GL) contrast to perform
 240 S-DIC measurements (Fig 3). The left-top part is left transparent, but a
 241 thin tissue paper is interleaved between the projectile and the sample, to ob-
 242 serve the formation of crack patterns on a uniform background (masking the
 243 background scene that makes cracks more difficult to read). The interest of
 244 painting only one half of the plate is to allow for an easy association between
 245 the evolving cracking pattern of the laminated glass and the deformed surface

246 measured via S-DIC. Having access to these cracks evolution gives informa-
 247 tion on the time duration of the elastic deformation phase of the sample, the
 248 formation of the radial cracks, orthoradial ones, the beginning of the tearing
 249 phase and the perforation step for instance. The S-DIC measurements can
 250 also be extrapolated on the non-speckled area of the plate, and therefore a
 251 measurement of the cracks evolution via the residuals is made possible on
 the reference surface. Table 1 summarizes all the extrinsic DIC parameters.

Cameras	Photron [®] SA-5
Definition	640 × 376 pixels
Gray Levels rendering	8 bits
Lens	Nikon AF Nikkor 24mm f/2.8D
Aperture	f/11
Field of view	500 mm × 300 mm
Image scale	≈ 0.7 mm/px (in the center)
Stereo-angle	right camera: 33°, left camera: 37°
Stand-off distance	between 70 and 80 cm
Image acquisition rate	30,000 Hz
Patterning technique	white dots over black surface
Pattern feature size	in the range of 3 to 10 pixels

Table 1: DIC hardware parameters

252

253 A trustworthy calibration of stereo-camera system is essential to perform
 254 a 3D kinematic measurement during the dynamic testing. Therefore the
 255 calibration procedure is further explained in [Appendix A](#).

256 3. Axisymmetric S-DIC analysis

257 Stereo-Digital Image Correlation (S-DIC) constitutes a technique of
258 choice to measure, on the one hand, the plate global and time resolved de-
259 formation and, on the other hand, the crack pattern evolution, which leads
260 to the estimation of PVB stretching at the glass plies cracks.

261 In the experiment, the laminated glass deformation is monitored by the
262 two lateral cameras. The out-of-plane displacement field is obtained from a
263 S-DIC analysis (using Correli 3.0 software (Leclerc et al. 2015)) during the
264 first 3 ms (*i.e.*, 90 images) of the experiment. During this period of time,
265 global S-DIC can be performed over the speckle surface: the elastomeric
266 paint remains elastic and only a few radial cracks can be observed on the
267 DIC residuals. A change in brightness takes place on the deformed surface
268 because of light reflection as the impactor penetrates into the laminate.

269 3.1. Global S-DIC procedure

270 The global S-DIC procedure and the used notations are shortly recalled
271 thereafter and detailed in Appendix B and interested readers can refer to
272 previous publications (Hild and Roux 2012; Dufour et al. 2015b; Berny 2020)
273 for a more extended presentation.

274 The specimen kinematics is described in 3D by a displacement field $\mathbf{U}(\mathbf{X})$
275 such that a material point \mathbf{X} in the reference configuration moves to a new
276 position $\mathbf{X}' = \mathbf{X} + \mathbf{U}(\mathbf{X})$. For each camera, labeled c , the projective model
277 allows relating the 3D point to its position in the camera image plane, $s\mathbf{x}^c =$
278 $[\mathbf{\Pi}]^c \mathbf{X}$, and hence its movement obeys $s\mathbf{x}'^c = s(\mathbf{x}^c + \mathbf{u}^c) = [\mathbf{\Pi}]^c(\mathbf{X} + \mathbf{U}(\mathbf{X}))$.
279 Grey level conservation between the reference image f^c (captured by camera

280 c) and the deformed one g^c (from the same camera) is a basic assumption of
 281 Digital Image Correlation that leads to the determination of the kinematic
 282 as the minimization of the L2 norm of the so-called residual field

$$\varrho^c(\mathbf{X}, \mathbf{U}(\mathbf{X})) = [g^c(\mathbf{x}'^c) - f^c(\mathbf{x}^c)] \quad (1)$$

283 This minimization is performed based on a parametrization of the kinematics
 284 \mathbf{U} , chosen here as supported by a 3D finite element mesh, namely $\mathbf{U}(\mathbf{X}) =$
 285 $\sum_i U_i \theta_i(\mathbf{X})$ where θ_i is the shape function vector associated to node i , and
 286 U_i the nodal amplitude. These nodal displacements gathered into a vector
 287 $\{\mathbf{U}\}$ are computed using a Gauss-Newton algorithm, namely progressively
 288 correcting the displacement with a correction vector $\{\delta\mathbf{U}\}$, solution of the
 289 tangent problem

$$\sum_{c=1}^{N_c} [\mathbf{H}]^c \cdot \{\delta\mathbf{U}\} = \sum_{c=1}^{N_c} \{\mathbf{b}\}^c \quad (2)$$

290 where the Hessian $[\mathbf{H}]^c$ and second member $\{\mathbf{b}\}^c$ are sums over all cameras
 291 c of individual contributions

$$\begin{aligned} H_{ij}^c &= \sum_{\mathbf{x} \in \text{ROI}} \left[\frac{\partial \mathbf{u}^c}{\partial U_i} \cdot \nabla f^c(\mathbf{x}^c) \right] \left[\frac{\partial \mathbf{u}^c}{\partial U_j} \cdot \nabla f^c(\mathbf{x}^c) \right] \\ b_i^c &= \sum_{\mathbf{x} \in \text{ROI}} \left[\frac{\partial \mathbf{u}^c}{\partial U_i} \cdot \nabla f^c(\mathbf{x}^c) \right] [f^c(\mathbf{x}^c) - g^c(\mathbf{x}'^c)] \end{aligned} \quad (3)$$

292 The S-DIC problem is intrinsically an ill-conditioned (if not ill-posed)
 293 problem. The use of Tikhonov regularization improves the numerical con-
 294 ditioning of the problem, but at the expense of diverting the solution from
 295 reality because it is a numerical ‘trick’ whose physical justification is ill-
 296 founded. As a consequence, it may lead to artifacts when for instance large
 297 rotations occur (Colantonio et al. 2020). The ideal solution would be to help

298 the problem formulation using a mechanical modeling of the laminated glass
 299 response. However, the fragmentation of the glass plies, the delamination
 300 of the PVB and its extension involving significant geometric non-linearities
 301 (large displacements and rotations) makes this route quite challenging. Alter-
 302 natively, the axisymmetry of the set-up is a very strong constraint which may
 303 be used for regularization without any prior knowledge or fragile assumption
 304 about the mechanical model of the monitored sample, even if this symme-
 305 try is expected to be broken locally in the long run as the laminated glass
 is fragmented, and finally torn apart. For global S-DIC, the T3-mesh pre-

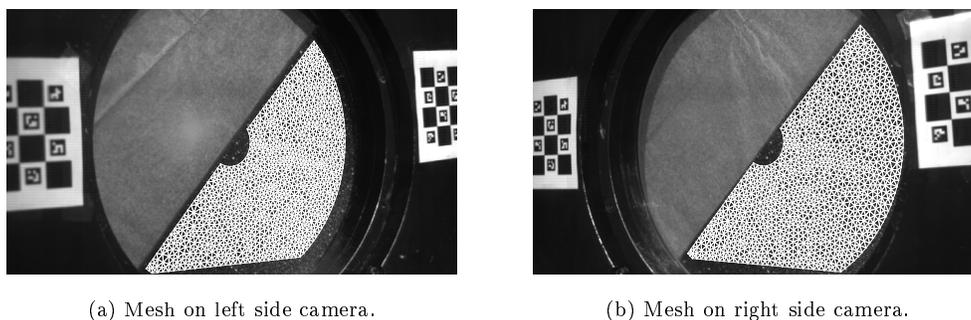


Fig 4: Initial 2D mesh projected on the 2 reference frames. All are T3 elements of 4 mm average length.

306
 307 sented 4 mm length elements that are superimposed on the reference frames
 308 on Fig 4. There is a total of 2366 elements and each element possesses 32
 309 integration points, so that all gray level information on the images is consid-
 310 ered during S-DIC calculation. S-DIC parameters are mentioned in table 2.
 311

DIC software	Correli 3.0 (Leclerc et al. 2015)
Element sizes	4 mm (≈ 12 px)
Number of elements	2366
Shape functions	Linear (T3 elements)
Evaluation points (per element)	32
Displacement noise-floor	≤ 40 μm

Table 2: Global S-DIC parameters

312 *3.2. Axisymmetric S-DIC*

313 A new global axisymmetric description of the displacement field can be
314 inserted as a constraint for the minimization problem. The axisymmetric
315 based regularization proposed thereafter is a non-invasive constraint that
316 resumes the unknown displacement field (3D vector field over a 2D surface)
317 to a much simpler one (2D displacement field over a 1D radial profile). Such
318 a regularization could be done with B-splines for instance ([Chapelier et al.](#)
319 [2021](#)), but a piece-wise linear description will be adopted in the following. It
320 can be seen as a reduction of the degrees of freedom of the initial mesh nodal
321 displacements.

322 Expressed in cylindrical coordinates, the displacement between two im-
323 ages is a purely radial and out-of-plane displacement

$$\mathbf{U}^{\text{ax}}(r) = [\mathbf{U}_r(r), 0, \mathbf{U}_Z(r)]$$

324 The radius being directly $r_i = \sqrt{X_i^2 + Y_i^2}$ and the angle $\alpha_i = \arctan(Y_i/X_i)$.

325 The orthonormal displacement is enforced to be globally null, which re-
326 mains a reasonable assumption as long as no tear opens up in the interlayer

327 (a phenomenon, local at first, that propagates and breaks the axisymmetry of
 328 the deformation even at a global scale). Radial and axial displacements only
 329 depend on the radial distance r between the impact location and the nodal
 330 position. The FE mesh center is localized at the impact initial position, *i.e.*,
 331 the center of the laminated glass plate, with a 200 μm uncertainty.

332 The transformation from cartesian to cylindrical coordinates is written
 333 as a rotation matrix $[\mathbf{P}]$, combining the smaller transfer matrices \mathbf{P}_i for each
 334 node i

$$\mathbf{P}_i = \begin{bmatrix} \cos \alpha_i & -\sin \alpha_i & 0 \\ \sin \alpha_i & \cos \alpha_i & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (4)$$

335 such as

$$\mathbf{U}(\mathbf{X}_i) = \mathbf{P}_i \mathbf{U}^{\text{ax}}(r_i) \quad (5)$$

336 The axisymmetric description will be enforced via the introduction of 1D-
 337 linear shape functions (Fig 5), limiting the displacement measurement along
 338 r and Z directions. The 2D-mesh current nodal position (represented by
 339 circles) are represented on the new shape functions.

340 In addition, rigid modes constraining all the nodes in respectively X , Y
 341 and Z directions, are added to possibly compensate the experimental Z -axis
 342 location error during the axisymmetric S-DIC procedure. In other words,
 343 the addition of those three additional modes allows to shift and correct the
 344 symmetry axis location.

345 The 1D shape functions are noted $q_i(r)$ and filled in a second transfer

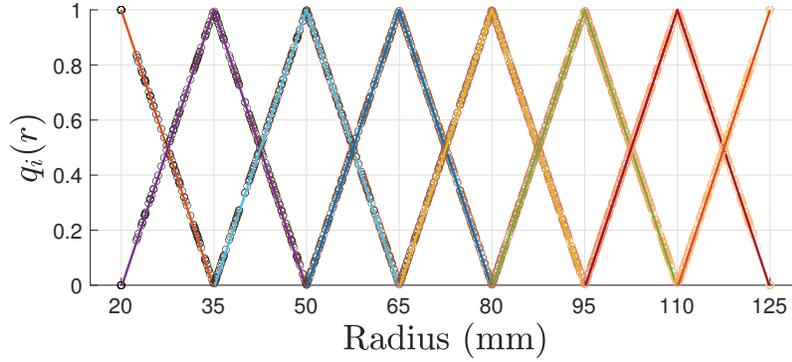


Fig 5: 1D shape functions $q_i(r)$ insuring displacement continuity, in straight lines. Interpolated T3-nodes (Fig 4), as circles on the new shape functions. For better readability the plot illustrates 15 mm-long elements, while 4 mm elements were actually used in the DIC measurements.

346 matrix $[\mathbf{Q}]$ such as the new Hessian and right-hand terms become

$$\begin{aligned} [\mathbf{H}]^{ax} &= ([\mathbf{P}][\mathbf{Q}])^\top \cdot \left(\sum_{c=1}^{N_c} [\mathbf{H}]^c \right) \cdot [\mathbf{P}][\mathbf{Q}] \\ \{\mathbf{b}\}^{ax} &= ([\mathbf{P}][\mathbf{Q}])^\top \cdot \left(\sum_{c=1}^{N_c} \{\mathbf{b}\}^c \right) \end{aligned} \quad (6)$$

347 and the minimization problem

$$[\mathbf{H}]^{ax} \cdot \{\delta\mathbf{U}\}^{ax} = \{\mathbf{b}\}^{ax} \quad (7)$$

348 The global 3D-displacement increment $\{\delta\mathbf{U}\}$ is updated via the two transfer
349 matrices and the nodal axisymmetric displacement $\{\delta\mathbf{U}\}^{ax}$

$$\{\delta\mathbf{U}\} = [\mathbf{P}][\mathbf{Q}] \cdot \{\delta\mathbf{U}\}^{ax} \quad (8)$$

350 This newly introduced reduced problem is far better conditioned than the
351 original S-DIC one (decreasing the degrees of freedom from 7098 to 84).

352 Therefore no additional regularization is needed to perform axisymmetric
353 S-DIC.

354 4. Results and discussion

355 Global S-DIC and axisymmetric S-DIC have been performed on the
356 painted deformed surface of the laminated glass, during the first 3 millise-
357 conds following impact, or a total of 90 frames per camera. Results at different
358 times are shown in figures 6, 7 and 8, corresponding respectively to the 20th,
359 50th and 80th frames. Both raw camera images, (a) left view and (b) right
360 view, are represented in gray levels. Then the left column corresponds to
361 the global S-DIC analysis and the right column to the axisymmetric S-DIC
362 analysis, with: (c)+(d) the measured residuals by the left-side camera (im-
363 age (a)), (e)+(f) the measured residuals by the right-side camera (image (b)),
364 (g)+(h) the out-of-plane displacement field on the reference mesh and (i)+(j)
365 the current deformed 3D surface.

366 A crushed glass zone, centered at the impact point appears as a bright
367 white zone in Fig 6a-b. The formation of radial and orthoradial cracks could
368 be observed during the whole experiment. First, radial cracks propagate
369 from the impact point to the edge, so that fragment sizes increase with the
370 distance from the center. Then orthoradial cracks appear, as can already be
371 seen in Fig 6a-b. As time passes, the density of both radial and orthoradial
372 cracks increases, leading to smaller and smaller fragments.

373 This growing crack number also increases the deformability of the frag-
374 mented laminated glass plate, whose glass shards are still connected by in-
375 terlayer polymeric bridges that sustain large strains.

376 The geometry of the deformed plate surface could at first be approximated
377 by a rounded cone of increasing height. This information could also be read
378 on the displacement fields (g-j) for the three figures 6, 7 and 8.

379 The sample shape evolution and fragment orientation induce some light
380 reflection, with severe gray level changes on the captured images. Specular
381 reflections are even observed in some images, (such as the white spot at the
382 top of Fig 8a), making the analysis more and more difficult for the S-DIC
383 software.

384 A brightness change, not being accounted for by the S-DIC procedure,
385 is finally present in the residual field Fig 6(c-f). In particular, a contrast in
386 the residual field across a radial boundary, may be interpreted as a localized
387 bending along a radial crack, making some of them apparent although they
388 could hardly be perceived in the raw images.

389 In addition, due to the change in brightness in the right-hand zone, very
390 large (erroneous) displacements are obtained at some isolated nodes (see
391 Fig 8i) because of the extreme violation of the gray level conservation as-
392 sumption. This kind of displacement is however well filtered with the ax-
393 isymmetric regularized displacement (Fig 8j).

394 In Fig 7(c-f) and even more in Fig 8(c-f) axisymmetric regularization
395 gives rise to a larger rms residual, and yet the displacements are very precisely
396 equal to the trustful part of the non-regularized S-DIC computation. Indeed,
397 each local non-axisymmetric displacement is correctly accounted for in the
398 axisymmetric regularized version, without altering the measurement.

399 The cracks are easier to identify on the residual fields (c-f) for advanced
400 steps of the test as stretching and delamination of the PVB bridges increase.

401 The separation between two fragments split by a crack may reach about one
402 millimeter, and becomes very clearly visible on the DIC residuals as the pixel
403 size is approximately 1 mm.

404 Accessing the final crack pattern by an examination of the post-fractured
405 sample in fragmentation experiment is not easy as many non-reversible phe-
406 nomena occur: large stretching then tearing of the interlayer, delamination
407 between the PVB and the glass shards, comminution at the center of the plate
408 that loses smaller fragments sometimes crushed into glass dust. Therefore the
409 glass damage is hard to qualify, evolving throughout the whole test duration.
410 Three pictures have been taken at the end of this experiment (Fig 9). As can
411 be seen, the PVB layer has been torn and glass fragments are missing close to
412 the impact location. Farther along the radial direction, the laminated glass
413 retains a crater-like or cone-like shape at the end of the experiment which
414 emphasizes a residual irreversible deformation of the plate. The failure mode
415 encountered during this test can be assimilated to punching failure (Li et al.
416 2023) as the area under the impactor is very finely fragmented and the tear-
417 ing occurred at 40 mm – very close – to the center of the plate where glass
418 fragments detached from the adhesive layer. Large fragments areas, located
419 at more than 100 mm from the impact, may still sustain secondary impacts
420 as the local delamination and PVB stretching has been very low.

421 Many cracks can be observed on these three images, thanks to a good
422 spatial resolution, but the temporal damage qualification cannot be assessed
423 via post-fractured imaging. In the following is proposed a live monitoring of
424 the crack formation to complete the damage analysis.

425 Due to the global axisymmetry of the measured displacement field, it is

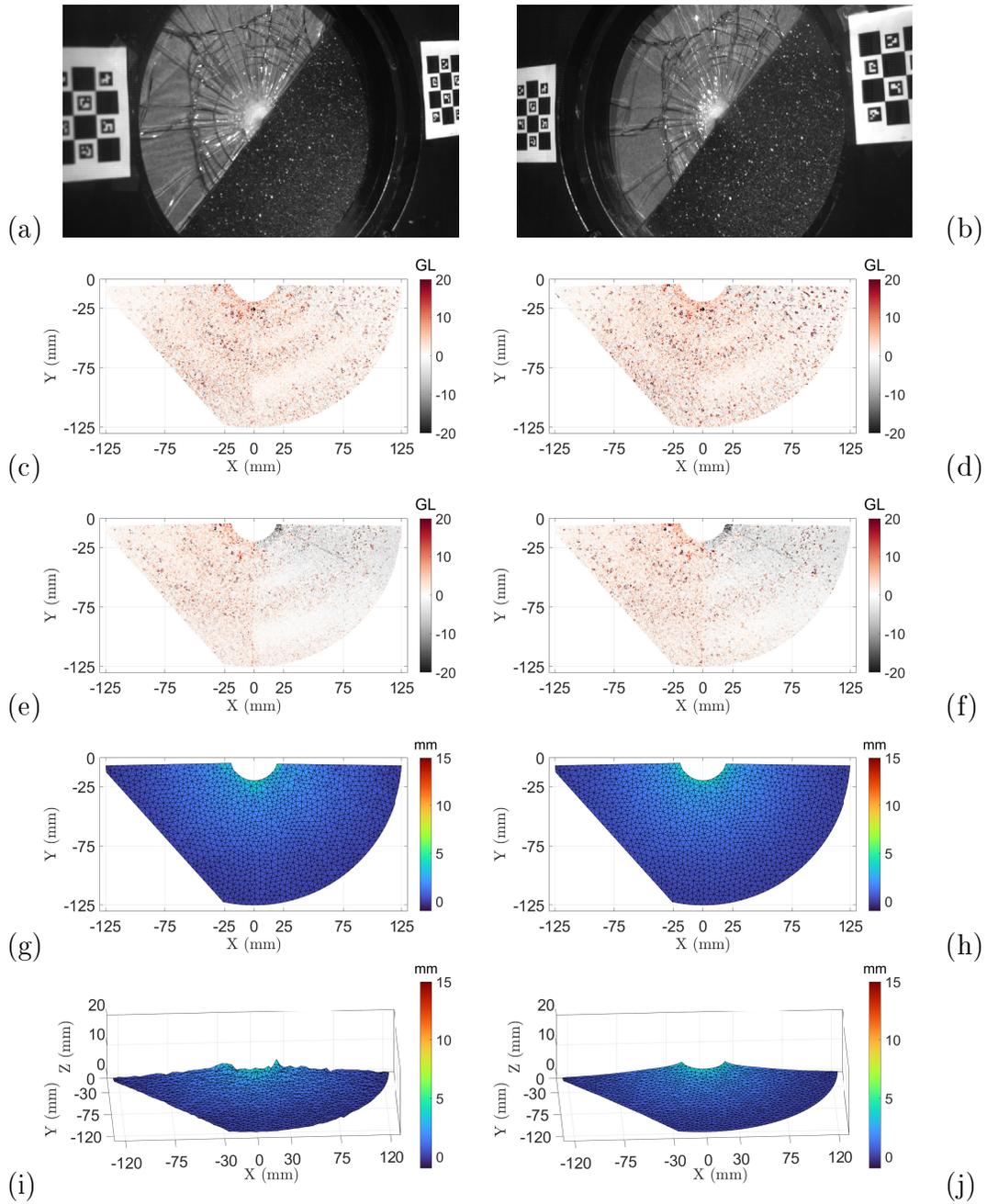


Fig 6: S-DIC measurement at $t=0.63$ ms (*i.e.*, iteration 20): (a)+(b) Stereo images. Corresponding residual fields at convergence (in gray levels): (c)+(e) S-DIC (d)+(f) Axisymmetric. The out-of-plane displacement field on initial mesh (mm): (g) S-DIC (h) Axisymmetric. Idem on deformed mesh (mm): (i) S-DIC (j) Axisymmetric.

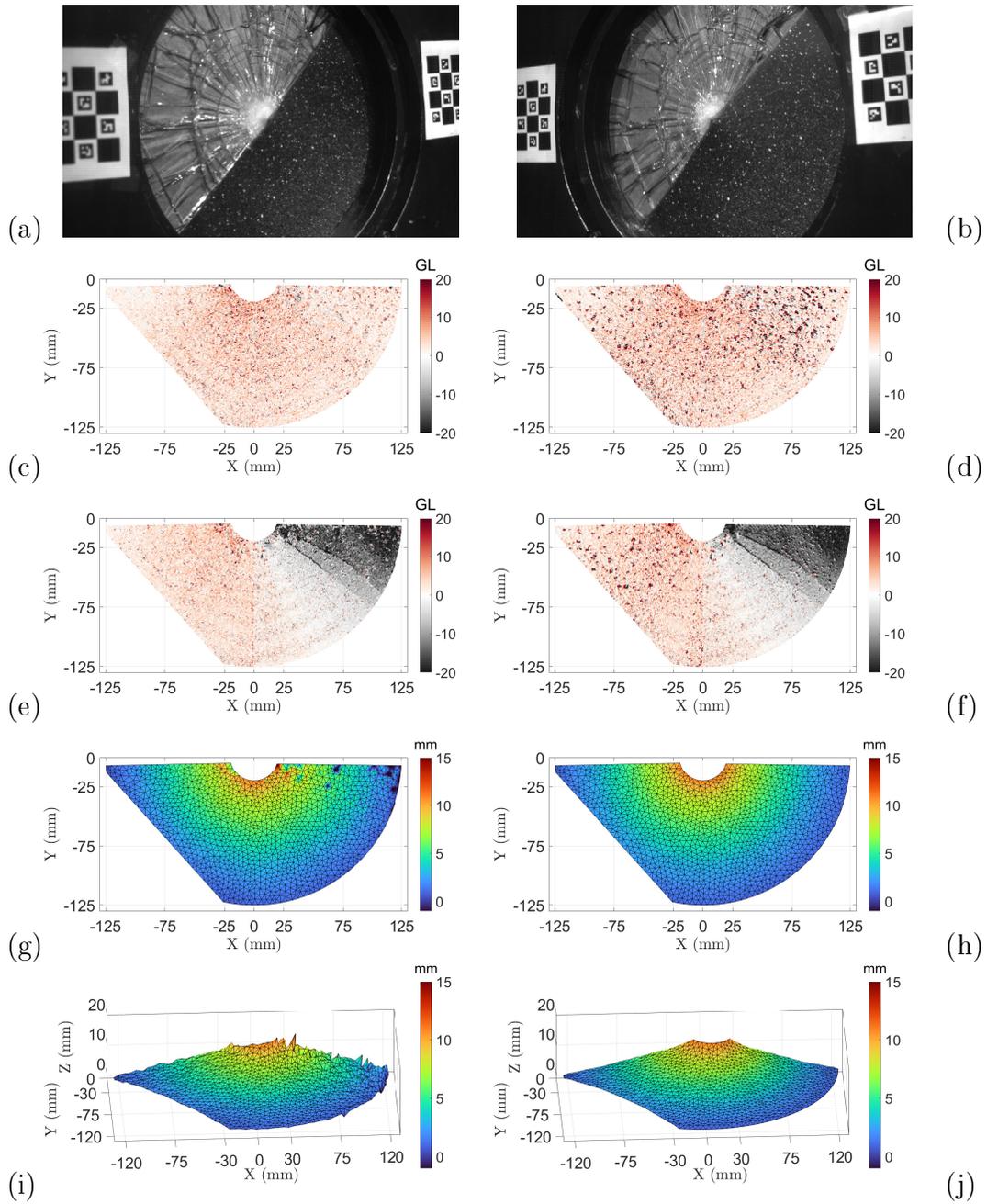


Fig 7: S-DIC measurement at $t=1.63$ ms (*i.e.*, iteration 50): (a)+(b) Stereo images. Corresponding residual fields at convergence (in gray levels): (c)+(e) S-DIC (d)+(f) Axisymmetric. The out-of-plane displacement field on initial mesh (mm): (g) S-DIC (h) Axisymmetric. Idem on deformed mesh (mm): (i) S-DIC (j) Axisymmetric.

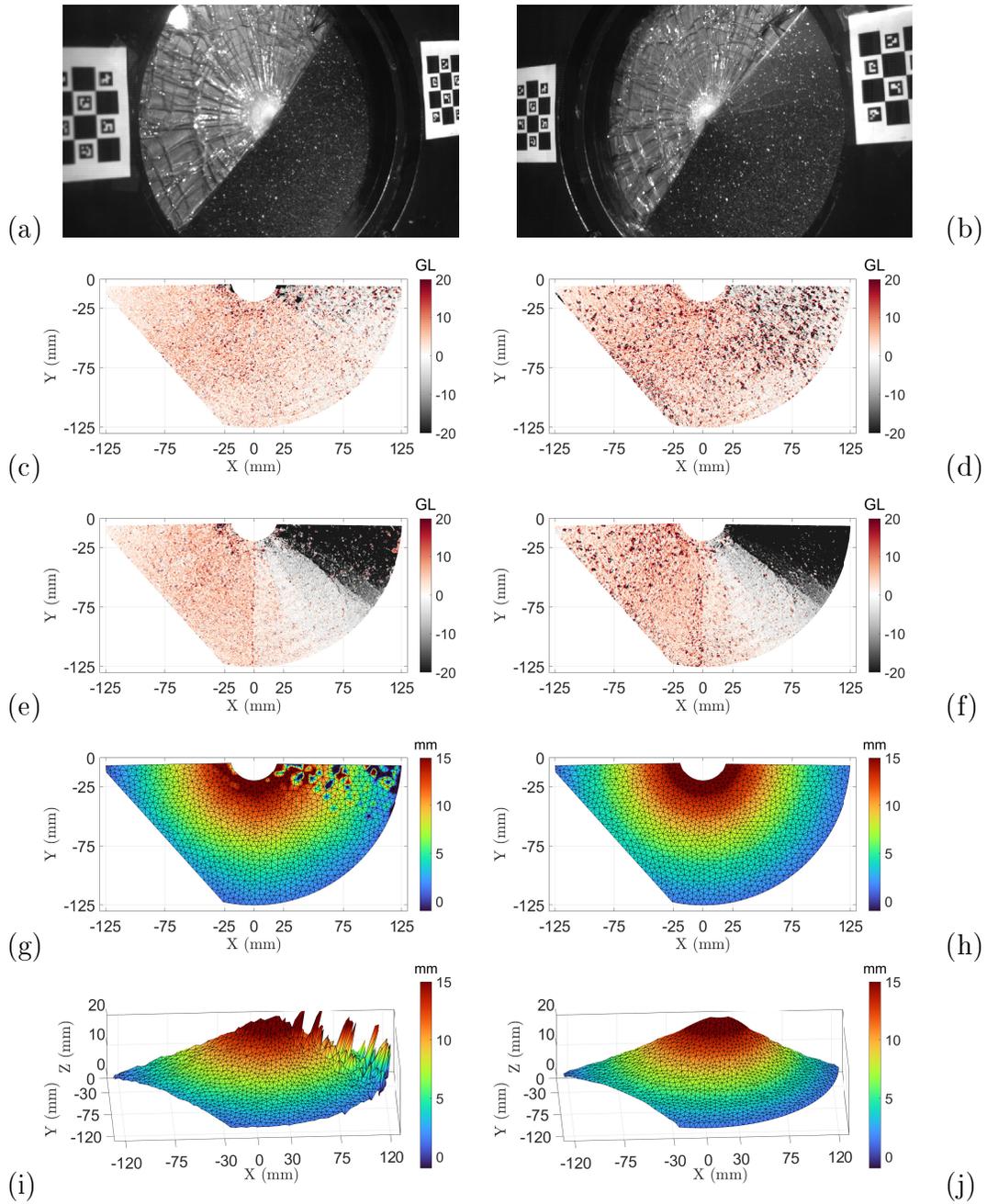


Fig 8: S-DIC measurement at $t=2.63$ ms (*i.e.*, iteration 80): (a)+(b) Stereo images. Corresponding residual fields at convergence (in gray levels): (c)+(e) S-DIC (d)+(f) Axisymmetric. The out-of-plane displacement field on initial mesh (mm): (g) S-DIC (h) Axisymmetric. Idem on deformed mesh (mm): (i) S-DIC (j) Axisymmetric.

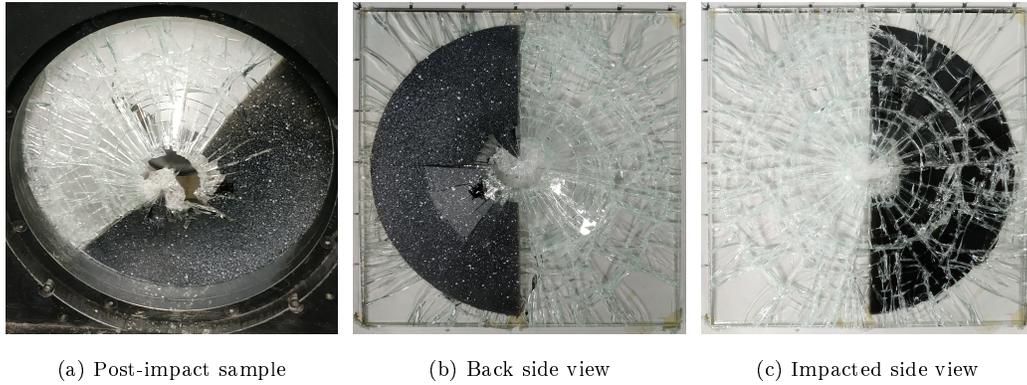


Fig 9: Post-breakage sample observations performed just after the experiment: (a) sample is still set in the device. Flat positions: (b) Back side or monitored side view, (c) Front side or impacted side view.

426 possible to estimate the deformation of the non-speckled glass surface on
 427 a semi circular mesh, then analyze the residuals between a reference non
 428 fragmented image and the experimental deformed image in the Lagrangian
 429 frame. Some residuals of the non-speckled part are represented on Fig 10.
 430 The highest gray level value residuals correspond to cracks location in the
 431 glass plies. It is possible to follow their length throughout the experiment.
 432 First, on Fig 10a-b, radial cracks are growing from the impact location to-
 433 ward the sample edges. Intense radial stresses initiate a few first orthoradial
 434 (circular) cracks, for example in Fig 10a, at coordinates (25, 75) and (-40,
 435 50) millimeters. An area between 60 mm to 90 mm radius (Fig 10c-j), con-
 436 centrates many orthoradial cracks, and delimits the finer fragmented area at
 437 the center of the plate from the larger fragmented zone at higher radius. This
 438 circular boundary was highlighted in Nourry's thesis and drives the global
 439 response of the laminated glass subjected to blunt impact (Nourry 2005).

440 Some limits can be mentioned in terms of crack measurement accuracy.

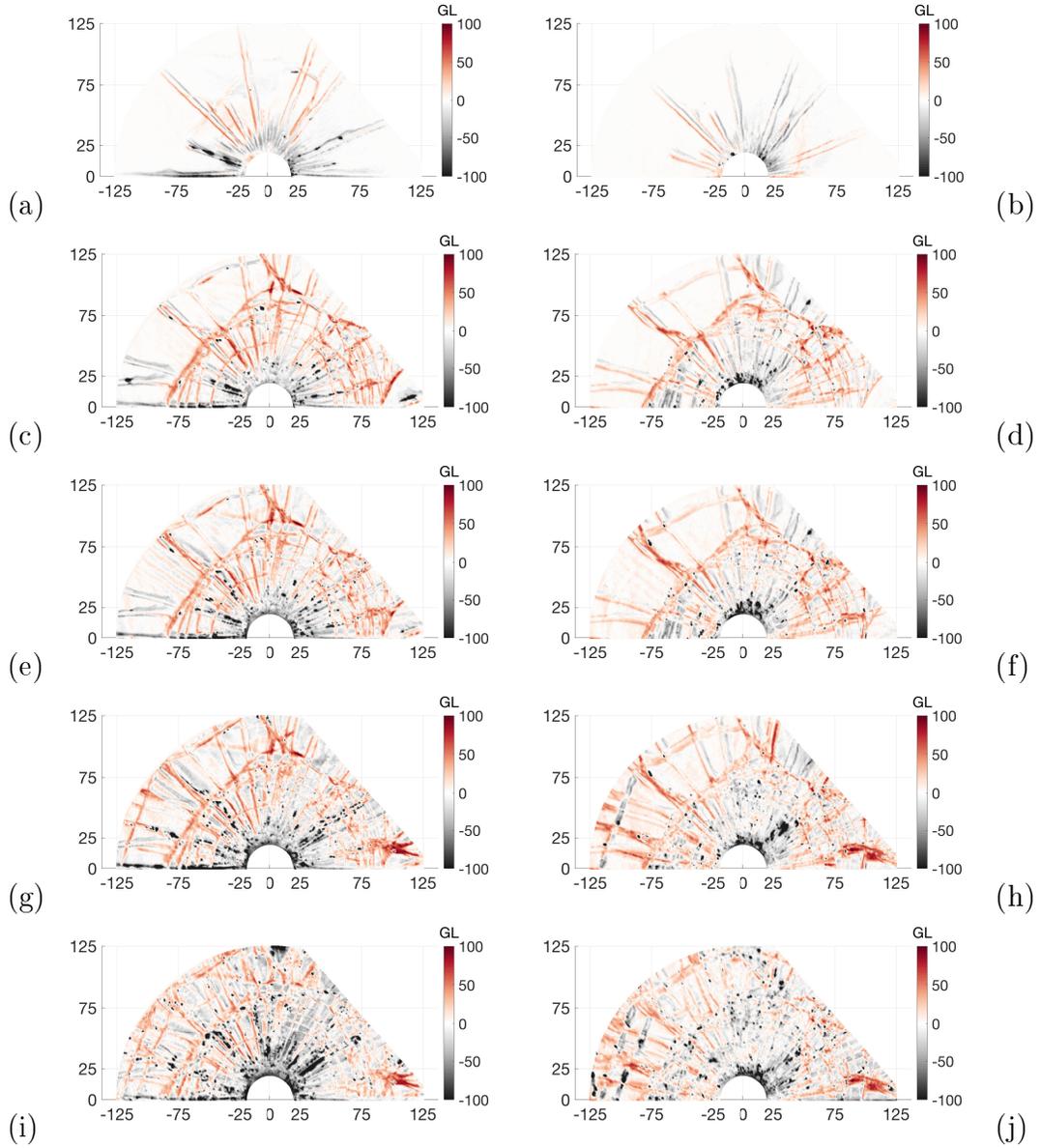


Fig 10: Residuals of the non-speckled area of the glass surface, projected on $Z=0$ mm plane. (a)-(b) $t=0.40$ ms. (c)-(d) $t=0.63$ ms. (e)-(f) $t=1.13$ ms. (g)-(h) $t=1.63$ ms. (i)-(j) $t=2.63$ ms. Left and right columns corresponds respectively to the left-side and right-side camera.

441 The first one is the crack duplication on the residual maps, caused by their
442 projected shadows on tissue paper. This effect can be illustrated by the
443 measured residual map of Fig 10c (same instant than Fig 6a) where cracks
444 shadows can be seen. The effect might have been reduced using a darker
445 tissue paper as a uniform background. A second limitation is due to the
446 angular shift between the cameras and the device axis. This is enhanced by
447 the large stretching of the PVB links between glass fragments, and by the
448 widening cracks during the plate movement (see Fig 10j for instance). The
449 stack thickness is also a limitation, as a radial crack can propagate in the
450 first and second ply, and let a large imprint on the residuals. It can also be
451 noted that the 20 μ s temporal uncertainty in stereo-vision has to be adjusted
452 to measure a synchronized crack formation, as a crack pattern difference is
453 measured between Fig 10a and b.

454 A rapid increase of the S-DIC residuals is observed between frame 1 and
455 10 (Fig 11). Theses images correspond to the apparition of the radial cracks
456 and the first orthoradial cracks. Then a slower growth of the residuals is
457 observed up to frame 80. This evolution corresponds to the continuing for-
458 mation of cracks on the plate and the increasing out-of-plane displacement.
459 The larger contribution of the global residual ϱ is attributed to the change in
460 brightness and contrast, but the increasing number of nodes being attributed
461 an obviously incorrect displacement becomes dominant past image number
462 80. Therefore, from this point and on, the axisymmetric S-DIC global resid-
463 uals grow more slowly and actually become smaller than the regular S-DIC
464 ones.

465 The out-of-plane displacement field is globally well measured by S-DIC,

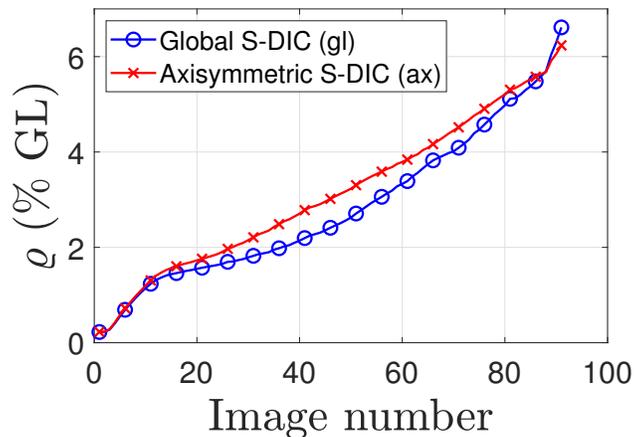


Fig 11: The global residual ρ evolution is represented as a function of image number.

466 although some nodes do not converge properly on the displacement fields
 467 (g)+(i) of Fig 7 and 8. Those were located at a small distance from the
 468 center of the mesh and in the very intense brightness change zones. Those
 469 erroneous measurements are cleaned up in the axisymmetric S-DIC measure-
 470 ment (h)+(j).

471 Moreover, an axisymmetric interpretation of the displacement fields is
 472 represented on Fig 12, interpolating the S-DIC obtained fields on radial shape
 473 functions (legend 'g'). The out-of-plane displacement is therefore filtered,
 474 and a good radial approximation of the displacement can be obtained reduc-
 475 ing the influence of the ill-behaved measurement points. The extrapolated
 476 displacement also matches the telemeter curve (black straight line), that can
 477 be interpreted as the displacement along the axis ($r = 0$), and fits the ax-
 478 isymmetric S-DIC results (legend 'ax'). The telemeter is used to map the
 479 deformation of the crushed zone where S-DIC cannot measure the kinematic
 480 fields due to very large discontinuities (Meyland et al. 2023). For large radius

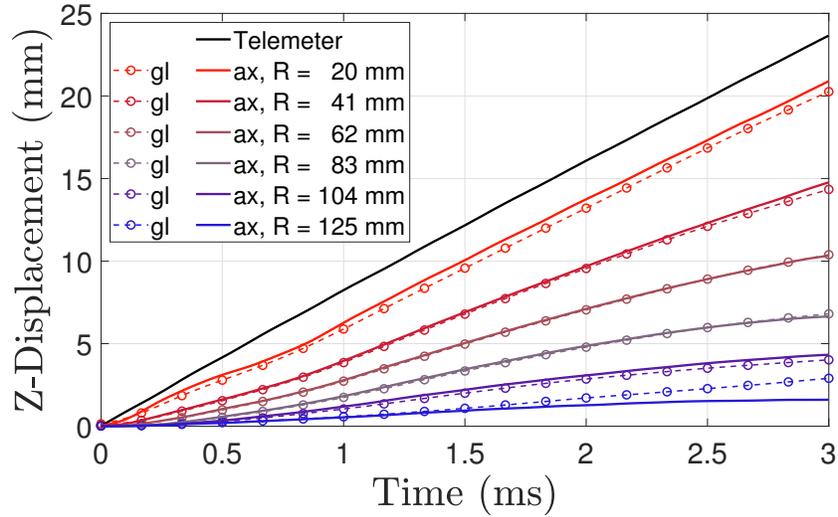


Fig 12: Out-of-plane displacement (mm) for some radius ('R', in mm) versus time (ms). Axisymmetric ('ax') and global ('gl') S-DIC are compared with the telemeter measurement.

481 and time, the global and axisymmetric results are moving away from each
 482 other showing that the axisymmetric code is more reliable than the initial
 483 global S-DIC procedure.

484 During the impact experiment, two areas can be distinguished: the first
 485 one right under the impactor head that which is finely fragmented, subjected
 486 to intense shear stress, quickly prone to tearing, and thus not able to dissi-
 487 pate a large amount of impact energy. The second area, starting from about
 488 70 mm from the plate center, consists of glass fragments – connected via
 489 polymeric bridging – where the interlayer is still able to sustain large strains
 490 and delamination from the glass without introducing a significant bending of
 491 the glass plies and shearing of the interlayer. Thanks to the S-DIC measure-
 492 ments the strain rate was estimated to 5 to 10 s⁻¹ that matches the order of
 493 magnitude of the membrane behavior – allowing delamination and stretching

494 – as introduced by Hooper (Hooper 2011). This membrane behavior is also
495 validated by authors as the out-of-plane displacement exceeds the laminated
496 glass thickness during the test (Angelides et al. 2019). The energy dissipa-
497 tion that occurs during an impact (such as the *cannon* experiment) through
498 delamination and stretching of a polymer layer can therefore be estimated
499 using the *through-crack tensile* (*TCT*) test, widely documented in the litera-
500 ture (Seshadri et al. 2002; Elzière 2016; Del Linz et al. 2017; Samieian et al.
501 2018; Fourton et al. 2020; Ma et al. 2022). This tensile test is performed on
502 two pieces of laminated glass connected by a PVB ligament whose delami-
503 nation front and stretching evolve. It has been pushed to cover a very large
504 range of velocities from quasi-static regimes up to 10 m/s (Del Linz et al.
505 2017), matching the *cannon* velocity range. Therefore, the link between the
506 interlayer stretching and its delamination from the glass shards is already
507 well documented. Thanks to the kinematic field and the cracking pattern
508 measurements presented in this article, it is possible to consider a model us-
509 ing the measured data with the *TCT* to describe the correct behavior of a
510 fragmented laminated glass plate under a hard-body blunt impact during a
511 *cannon* test. Those data are of interest to the experimenter to determine the
512 behavior of different areas of the laminate, rather than relying only on either
513 a local measurement via strain gauges or measurements on the impactor,
514 that do not give the whole plate kinematics. Each laminated glass fragment
515 has its own specific movement during an impact that can be interpreted as
516 a local *TCT* test. In a nutshell, these measurements, coupled with a correct
517 corresponding model of the fragmented laminated glass plate, should allow
518 the evaluation of the local dissipated impact energy at the glass fragments

519 edges and, after summation over all cracks, the evaluation of the total energy
520 absorbed by the broken laminated glass plate.

521 **5. Conclusion**

522 This paper studied a hard-body impact on a laminated glass using stereo-
523 digital image correlation with high-speed cameras. During the fragmentation
524 test, S-DIC analyses are carried out to extract the 3D shape variation and
525 the corresponding kinematic information of the laminated glass.

526 The S-DIC algorithm demonstrates its versatility in obtaining 3D shape
527 and field measurements. Using an FE-based mesh as a numerical model,
528 it is possible to update and deform the virtual model to seamlessly fit the
529 3D shape variation, which allows the complex 3D kinematics to be resolved
530 even in the late stage of the impactor stroke (as in the case of the presented
531 fragmentation test on laminated glass). The newly developed axisymmet-
532 ric description of the displacement field in S-DIC, which can be seen as a
533 relevant geometric regularization devoid of other material-based mechanical
534 assumptions, provides a remarkable robustness to the analysis, with very
535 good convergence properties. The obtained performances are much greater
536 with the help of this regularization which avoids the divergence of some nodes
537 of the mesh and allows the crack formation to be measured on the deformed
538 non-speckled part of the laminated glass.

539 In the context of laminated glass under impact monitoring fragmenta-
540 tion and large strain regime, S-DIC using a global formulation, is a powerful
541 measurement technique, allowing access to the fragmented plate-deformed
542 surface and crack density over the region of interest. However, many hy-

543 hypotheses were set to finally measure satisfying kinematic fields, countering
544 the ill-posed DIC problem (that can add displacement error). The very ex-
545 treme experimental conditions: ephemeral flashes of light, varying light
546 intensity, cracks, and discontinuities formation, finite strain regimes. all con-
547 tribute to make this test a very challenging one for S-DIC. Nevertheless, the
548 proposed axisymmetric regularization allowed to overcome many of those
549 obstacles.

550 From the present analysis, some directions of progress can be identified
551 for future developments:

- 552 – Adding contrast and brightness correction should make the analysis
553 more robust
- 554 – Considering the image series as a whole for a suited spatio-temporal
555 analysis may further extend the domain of applicability of S-DIC
- 556 – Model reduction techniques, coupled to the latter spatio-temporal anal-
557 ysis may further accelerate the computations, and reduce the number
558 of global kinematic unknowns.

559 Those directions are currently being investigated, and open promising per-
560 spectives.

561 The data treatment presented here gives access to the shape of the lam-
562 inated glass during the impact, yielding the geometrical link between the
563 energy dissipation close to the cracks (energy per delaminated area that can
564 be measured on *TCT* experiments), the crack pattern evolution over the
565 whole plate, and the global energy effectively dissipated (the diminution of
566 kinetic energy lost by the impactor). This link will be exploited to build a

567 model that computes the energy that the laminated glass can absorb as a
 568 function of material parameters only, such as the glass-interlayer adherence or
 569 its large-deformation stress-strain behavior to be presented in a forthcoming
 570 publication.

571 **Appendix A. Calibration of the Stereo Vision Camera System**

572 Finding the correspondence between the 3D coordinates of known points
 573 in the specimen and their projected 2D coordinates in retinal images cap-
 574 tured by stereo cameras (2 cameras or more) is crucial for accurate 3D shape
 575 reconstruction. This is performed in a so-called ‘calibration stage’, which
 576 consists in identifying the projection matrix (*i.e.*, $[\mathbf{\Pi}]$) to bridge the 3D co-
 577 ordinates of points, $(X, Y, Z, 1)^\top$ in the lab frame, to their 2D projection,
 578 $(x, y, 1)^\top$ in the camera image plane.

$$\begin{pmatrix} sx \\ sy \\ s \end{pmatrix} = [\mathbf{\Pi}]^j \begin{pmatrix} X \\ Y \\ Z \\ 1 \end{pmatrix} = [\mathbf{K}][\mathbf{RT}] \begin{pmatrix} X \\ Y \\ Z \\ 1 \end{pmatrix} \quad (\text{A.1})$$

579 The projection matrix, $[\mathbf{\Pi}]$, can be decomposed into an intrinsic matrix, $[\mathbf{K}]$
 580 — which characterizes the intrinsic parameters of camera (optical center,
 581 focal distance and skewness) — and an extrinsic projection matrix, $[\mathbf{RT}]$,
 582 which gives the relative position of the camera and 3D object (translation
 583 and rotation). s is a scale factor.

584 The projection matrix $[\mathbf{\Pi}]$ of each camera is commonly measured via
 585 a calibration object (Besnard 2010; Dufour et al. 2015b; Berny 2020). The
 586 technique consists in monitoring this well-known object, whose geometry and

587 size have been independently measured and stored for instance as a 3D-mesh
 588 file, expressed with the world coordinates of the problem. Both, intrinsic
 589 and extrinsic matrices, are identified thanks to the presented ‘self-calibration’
 590 procedure.

591 A classical Point-to-Point initialization using natural features (*e.g.*, edge
 592 and corners), provides an initial guess of the projection matrix. Then, the
 593 projection matrices are further refined in the Global CAD/FE-based frame-
 594 work as proposed by (Dufour et al. 2015a; Shao et al. 2016; Berny 2020).
 595 Then comparing the gray level monitored by each camera at predefined in-
 596 tegration points location, the second phase of calibration provides a refined
 597 determination of the projection matrices (Berny 2020; Beaubier et al. 2012).

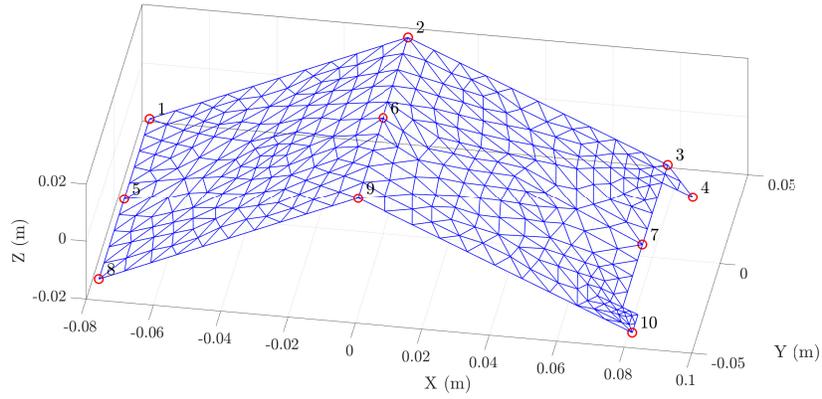
598

599 In this study, the calibration target is a dihedral (“open-book”) which has
 600 two square faces of 10 cm ($\pm 10 \mu\text{m}$) side oriented at 130° (Besnard 2010)
 601 (see Fig A.13b-e). An external support is printed to position the object at
 602 the center of the circular window. The 3D printer accuracy is estimated at
 603 $200 \mu\text{m}$.

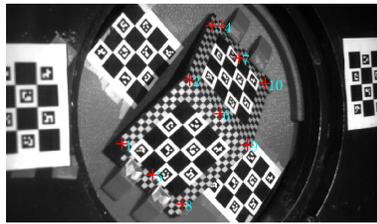
604 An FE mesh (Fig A.13a) is used throughout this paper as the virtual
 605 model of the calibration target, where $n = 1, \dots, N$ key points are selected
 606 in both the image (Fig A.13b-c) and virtual model (Fig A.13a) denoted as
 607 \mathbf{x}_n and \mathbf{X}_n respectively. The initialization of projection matrix, consists in
 608 minimizing the sum of quadratic differences

$$\mathcal{T} = \sum_{n=1}^N (\mathbf{x}_n - [\mathbf{\Pi}]\mathbf{X}_n)^2 \quad (\text{A.2})$$

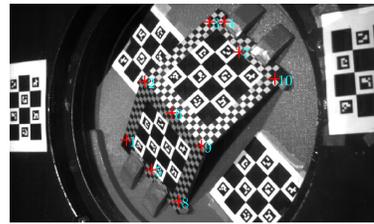
609 with respect to $\mathbf{\Pi}$. Initialization is validated by projecting the 3D mesh over



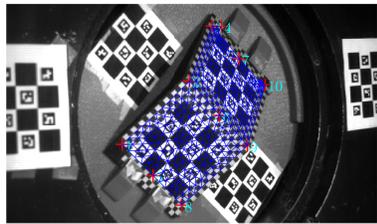
(a)



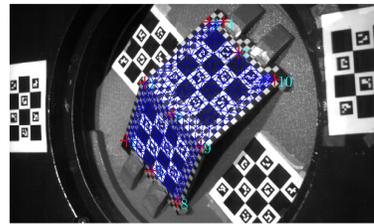
(b)



(c)



(d)



(e)

Fig A.13: Point-to-Point initialization:(a) Virtual FE model of calibration target; selected key points in FE model are plotted as red circles (b)-(c) Hand-selected key points in images for cameras 1 and 2 (d)-(e) Projected FE Mesh (plotted in blue) over the 2D image with the estimated projection matrices for cameras 1 and 2.

610 the 2D images for each camera (Fig [A.13d-e](#)).

611 Finally, as the geometrical relationship between the target and the speci-
612 men is well known, the surface of the laminated glass to be observed is easily
613 calibrated (see Fig [3](#))

614 **Appendix B. Global S-DIC method**

615 Digital Image Correlation most often relies on the assumption of gray
616 level conservation between a reference image f and a deformed image g , so
617 that for each pixel \mathbf{x}

$$f(\mathbf{x}, t = 0) = g(\mathbf{x} + \mathbf{u}(\mathbf{x}), t) \tag{B.1}$$

618 where t is the time and \mathbf{u} the pixel displacement. However, this equation is
619 never strictly obeyed due to the presence of acquisition noise, dependence on
620 the camera performances, or subpixel gray-level interpolation error and hence
621 the displacement field is computed from the minimization of the quadratic
622 difference between the two members of Eq. (B.1).

$$\mathcal{W}(\mathbf{u}) = \|f(\cdot) - g(\cdot + \mathbf{u})\|^2 \tag{B.2}$$

623 calculated on all pixels of the region of interest (ROI).

624 The philosophy is similar for global S-DIC with multiple view system,
625 but minimization is performed for the 3D displacements and considering all
626 the N_c cameras at once (Hild and Roux 2012; Dufour et al. 2015b; Berny
627 2020) (in our case $N_c = 2$). The minimization problem, described in equa-
628 tion (2), finally consists of successive corrections of the displacement field
629 from a system assembled from the sum over all cameras of their contribution
630 to the global residual of global stereo-Digital Image Correlation. It is worth
631 emphasizing that the measured displacement is directly the 3D nodal dis-
632 placement field, minimizing the camera contribution (*i.e.*, residuals for each
633 camera) simultaneously.

634 The S-DIC measurements presented thereafter are also regularized by a
635 Tikhonov regularization to solve this poorly conditioned problem (Tikhonov

636 and Arsenin 1977). The minimization problem becomes

$$\left(\sum_{c=1}^{N_c} [\mathbf{H}]^c + \lambda [\mathbf{I}] \right) \cdot \{\delta \mathbf{U}\} = \sum_{c=1}^{N_c} \{\mathbf{b}\}^c + \lambda (\{\mathbf{U}_0\} - \{\mathbf{U}\}) \quad (\text{B.3})$$

637 with $[\mathbf{I}]$ the identity matrix, \mathbf{U} the current corrected displacement and \mathbf{U}_0
638 the estimated displacement for this iteration. The weight λ associated to the
639 Tikhonov regularization is chosen as

$$\lambda = 10^{-2} \cdot \lambda_{max} \quad (\text{B.4})$$

640 with λ_{max} the largest eigenvalue of the S-DIC Hessian matrix $\sum_{c=1}^{N_c} [\mathbf{H}]^c$.

641 As a remark, the Tikhonov regularization is no longer used when the
642 axisymmetric formulation (section 3) is chosen.

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